EMPIRICAL STUDY ON PRICING EFFICIENCY OF BLACK SCHOLES MODEL OF OPTION PRICING WITH SPECIAL REFERENCE TO EQUITY OPTIONS IN INDIA

DR. KAPIL ARORA  
PROFESSOR, ALLIANCE UNIVERSITY, BANGALORE

MR. MANISH SHARMA  
RESEARCH SCHOLAR, J K LAKSHMIPAT UNIVERSITY, JAIPUR; ASSISTANT PROFESSOR, JECRC UNIVERSITY, JAIPUR

ABSTRACT: This paper is intended to find the efficiency of the Option prices derived by using Black Scholes Option Pricing Model. Ten stocks are taken as sample for determination of BSOPM call and put option prices. For using BSOPM 7 days moving average prices are considered for the computation of volatility. It has been found that theoretical option prices are mispriced while compared with the actual market option prices. Thus, it is concluded that BSOPM model does not seem to be fit for pricing option contract and is not efficient for equity option pricing in Indian market. It is recommended that some other factors like volumes and implied volatility are also responsible for price changes in option contacts other the five predictors mentioned in the Black Scholes Model.

Keywords: BSOPM, Volatility, Option, Implied Volatility

INTRODUCTION: Derivatives are the contracts which are made on the basis of any underlying asset i.e. a security, an index, interest rate, currency etc. A derivative contract is arranged between two parties a buyer and a seller for a future point of time. There are different kinds of derivative contracts traded in India. Most of the trading takes place in futures, forwards and option contracts.

Forward contracts are customized contracts in which a buyer promised to buy certain quantity of an underlying at a certain price on a predefined future date from a seller. These contracts can be modified as per the consent of buyer and seller even after the contract is done. There is not interference and control of any intermediary in these contracts.

Future contract are standardized contract where exchange plays a role of intermediary between the buyer and the seller. The terms of the contracts are decided by the exchange and both buyer and seller will have to follow their norms, they cannot modify the norms as per their comfort. In future contract, both the parties are bound to fulfill their obligation on or before expiry.

Basically, Option Contract is a kind of derivative contract between two parties: a buyer and a seller, where the buyer of the option gets the right to buy or sell the underlying of that option contract; on the other hand the seller of the option has obligation to offset his position. For this, the buyer has to pay a certain amount- option premium to the seller of the option contract.

Option contracts can be classified into two categories: Call options and Put options. Call option provide right to buy whereas put option provide right to sell to the buyer of the option contract. The option seller has the obligation to offset his position. For buying an option
contract one has to pay some amount which is called premium of option contract. On the other hand option seller will receive the premium. The premium is made of two elements: intrinsic value and time value. Intrinsic value of the option is the spot price of an underlying beyond the strike price (higher in case of call and lower in case of put), whereas the time value of option is value that remains due to time to expire of option contract.

The price of an option contract is the premium that is acceptable to the seller of the option. There are several models which have been developed for the fair value pricing of option contracts. The Black and Scholes Model for option pricing is highly accepted model for pricing. The model of option pricing is based on the fundamental that in the future, the price of the underlying asset either increase or decrease as compare to the spot price of the underlying asset.

**Black Scholes Model for option pricing:**

The Black-Scholes model is used to calculate the theoretical price of European put and call options, ignoring any dividends paid during the option's lifetime. While the original Black-Scholes model did not take into consideration the effects of dividends paid during the life of the option, the model can be adapted to account for dividends by determining the ex-dividend date value of the underlying stock. The model makes certain assumptions, including:

- The options are European and can only be exercised at expiration
- Efficient markets (i.e., market movements cannot be predicted)
- No commissions
- The risk-free rate and volatility of the underlying are known and constant
- Follows a lognormal distribution; that is, returns on the underlying are normally distributed.

The Black-Scholes formula takes the following variables into consideration:

- Current underlying price
- Options strike price
- Time until expiration, expressed as a percent of a year
- Implied volatility
- Risk-free interest rates

\[ C = SN(d_1) - N(d_2)Ke^{-rt} \]

\[ d_1 = \frac{\ln(S/K) + (r + S^2/2)t}{\sqrt{t}} \]

\[ d_2 = d_2 - \sqrt{t} \]


The model is essentially divided into two parts: the first part, \(SN(d_1)\), multiplies the price by the change in the call premium in relation to a change in the underlying price. This part of the formula shows the expected benefit of purchasing the underlying outright. The second part, \(N(d_2)Ke^{(r-t)}\), provides the current value of paying the exercise price upon expiration (remember, the Black-Scholes model applies to European options that are exercisable only on
expiration day). The value of the option is calculated by taking the difference between the two parts, as shown in the equation. The put option can be calculated by using the following formula:

\[ P = -SN(-d_2) + N(-d_2)Ke^{-rt} \]

The following steps are used to calculate the theoretical price using BSOPM.

**Step I:** First, we calculate the historical volatility using the daily log returns by using moving average method.

- **Daily Return:** \( \ln(\text{today’s closing price}/\text{yesterday’s closing price}) \)
- **Daily standard deviation (SD):** \((\text{Variance of daily returns})^{0.5}\)
- **Historical Volatility:** \(\text{Daily SD} \times (250)^{0.5}\)

(Here we consider 250 trading days in a year)

**Step II:** We get all required value in the Black formula from the NSE website and use them in the Black Scholes model and we get the fair value of call and put options of various strike prices.

**Step III:** In the next step, we will find the differences between Model value and the actual market values.

After determination of Black Scholes Option pricing model we compare the model price data set with the actual price data set by using Paired sample test.

**REVIEW OF LITERATURE:**

Long & Officer (1997) investigated the relation between mispricing in the Black-Scholes option pricing model and volume in the option market. Their result indicated heavily traded call options are priced more efficiently and have lower mispricing errors than thinly traded options. On high trading days BSOP model mispricing errors were significantly larger than the normal working days. They inferred that large increase in volume might reflect new and changing marketplace information, thus making pricing less efficient in the BSOP model.

Kim et al. (1997) tested the strength of implied volatility estimates across option pricing models for at the money put options. They concluded that the implied volatility estimates recovered from the Black Scholes European Option Pricing Model was nearly identical from the implied volatility estimate obtained from Macmillan/Barone-Adesi and Whaley’s American put option pricing model. They also investigated whether the use Black Scholes implied volatility estimates in American put pricing model significantly affected the prediction of American put option prices.

Frino et al. (1999) tested the option pricing model using the historical data. Their conclusion was that the Black Scholes Model cannot be discarded. For the study, they conducted time series analysis of mispricing in order to determine whether that could be attributed to a market learning effect over time. They controlled the effect of dividend and possibility of early exercise and used to limit the possibility of incompatible risk free interest rate proxies having a confounding effect on results.

Gencay and Salih (2003) put light on the fact that the Black Scholes Pricing errors are more in the deeper out of the money options, and mispricing occurs more with the increased volatility. Their result indicated that the Black Scholes model is not a pricing tool in high volatility with considerably lower errors for out-of-the-money call and put options. They also mentioned that this could be invaluable information for practitioners as option pricing is a major challenge during high volatility periods.

Vasile et al. (2009) estimated the implied volatility and then studied the relationship between implicit volatility, moneyness and due term of options (volatility smile). Further, they tested the errors provided by Black Scholes model with respect to moneyness and due-term of options.
Angeli & Bonz (2010) examined the performance of Black Scholes model to price stock index options. They calculated the theoretical values of options under the Black-Scholes assumptions and compared these values with the real market prices in order to put the degree of deviation in two different time windows. They found clear evidence to state that BS Model performed different in the period before and after financial crisis.

Choi et al. (2011) determined implied volatility using the inverse function of Black Scholes Model and Least Square Support Vector Machine (LSSVM) model and found that LSSVM is more accurate than Black Scholes model since LSSVM’s MSE value is lesser than Black Scholes model’s MSE value. They used Hang Seng index option to verify the performances of these models.

Ray (2012) studied the Black Scholes Model of option pricing and made a more detailed analysis of the assumptions of the model and the mathematical derivation process of the model and also analysed the inherent loopholes in the theory.

Khan et al. (2013) incorporated modification in Black-Scholes option pricing model formula by adding some new variables on the basis of given assumption related to risk-free interest rate, and also showed the calculation process of new risk-free interest rate on the basis of modified variable. This paper also identified the various situations in empirical testing of modified and original Black-Scholes formula with respect to the market value on the basis of assumed and calculated risk-free interest rate.

Arora K. & Sharma M. (2013) determined the volatility and studied that how the implied volatility levels of an option contract of a stock is related to the pricing of that option and also determined whether a stock option is underpriced or overpriced. For this the sample data was collected from the stock options traded on the NSE. They used the basic statistical approach to determine the volatility of a stock and used this historical volatility in the Black-Scholes model in order to determine the implied volatility and then compared the historical volatility with the implied volatility to find whether an option is fairly priced.

Panduranga V. (2013) studied the significance of Black-Scholes model in Indian Derivative market with explicit reference to select cement stock options. Results of their study including paired sample T-test uncovered that there was no significant distinction between the expected option costs calculated thorough Black-Scholes Model and market cost of options. The study construed that model was relevant for the cement stocks.

**Objectives of Study:**

a). To figure the price of call and put option of ten diverse value shares.

b). To contrast the model prices and genuine market prices of call and put options and to test whether there is noteworthy distinction between the two informational collections.

**Research Methodology:**

Type of Research Design: Descriptive

Type of Data: Secondary Data

Time Duration: For study we have taken ten different stocks for the one year time duration from 1st January 2013 to 30th December 2013.
Sample Size:
The details of the data set are given as under:

<table>
<thead>
<tr>
<th>Scrip</th>
<th>Call</th>
<th>Put</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bharti Airtel</td>
<td>682</td>
<td>646</td>
<td>13970</td>
</tr>
<tr>
<td>HDFC Bank</td>
<td>769</td>
<td>605</td>
<td></td>
</tr>
<tr>
<td>HDFC</td>
<td>758</td>
<td>467</td>
<td></td>
</tr>
<tr>
<td>ICICI Bank</td>
<td>1112</td>
<td>924</td>
<td></td>
</tr>
<tr>
<td>Infosys</td>
<td>1118</td>
<td>575</td>
<td></td>
</tr>
<tr>
<td>ITC</td>
<td>741</td>
<td>633</td>
<td></td>
</tr>
<tr>
<td>L&amp;T</td>
<td>299</td>
<td>412</td>
<td></td>
</tr>
<tr>
<td>Reliance</td>
<td>670</td>
<td>503</td>
<td></td>
</tr>
<tr>
<td>Tata Motors</td>
<td>692</td>
<td>414</td>
<td></td>
</tr>
<tr>
<td>TCS</td>
<td>1062</td>
<td>888</td>
<td></td>
</tr>
<tr>
<td>SUM</td>
<td>7903</td>
<td>6067</td>
<td></td>
</tr>
</tbody>
</table>

DATA COLLECTION
We use the stock price data for ten stocks of S&P CNX Nifty for the period of 1st Jan 2013 to 31st Dec 2013 for the computation historical volatility (HV). HV is computed for year 2013 using 7 days moving average method. Further, we use the 91 days T bill rate yield (taken from the Reserve Bank of India website) as a proxy for risk free rate for using in Black Scholes model. By using this, we operationlise Black Scholes model and get the 7903 theoretical (model) values for call options and 6067 model values for put options. The actual option prices are picked from NSE website both for call and put options. These option contracts are basically cash settled European option contract (after 2013 only European contract are tradable in Indian option market).

TEST OF OPTION PRICING EFFICIENCY
The aim of the study is to test the pricing efficiency of BSOPM. For testing this BSOPM model are calculated and then compare with the actual prices by using the paired sample t test. To compute the theoretical prices (BSOPM model prices) we use the formula mentioned below:

\[ C = SN(d_1) - N(d_2)Ke^{-rt} \]
\[ d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{s^2}{2})t}{\frac{s}{\sqrt{t}}} \]
\[ d_2 = d_2 - \frac{s}{\sqrt{t}} \]

The formula to calculate the price of put options by using BSOM is as following:

\[ P = -SN(-d_1) + N(-d_2)Ke^{rt} \]

According to Black Scholes Model, the below mentioned variables are considered for option pricing:
- Current underlying price (S)
Options strike price (K)
Time until expiration, expressed as a percent of a year (t)
Volatility (s)
Risk-free interest rates (r)

The model is essentially divided into two parts: the first part, \( SN(d1) \), multiplies the price by the change in the call premium in relation to a change in the underlying price. This part of the formula shows the expected benefit of purchasing the underlying outright. The second part, \( N(d2)Ke^{-rt} \), provides the current value of paying the exercise price upon expiration (remember, the Black-Scholes model applies to European options that are exercisable only on expiration day). The value of the option is calculated by taking the difference between the two parts, as shown in the equation.

**Step I:** First, we calculate the historical volatility using the daily log returns by using 7 days moving average method.

Daily Return = \( \ln(\frac{\text{today's closing price}}{\text{yesterday's closing price}}) \)

Daily standard deviation (SD) = \((\text{Variance of daily returns})^{0.5}\)

Historical Volatility = Daily SD x \((250)^{0.5}\)

(Here we consider 250 trading days in a year)

**Step II:** We get all required value in the Black formula from the NSE website and use them in the Black Scholes model and we get the fair value of call and put options of various strike prices.

**Step III:** Now, We put the paired sample t test for finding whether there is significant difference between the theoretical price and actual option prices.

We use the following hypothesis for testing the difference between model and actual option prices:

Null Hypothesis (\( H_0 \)): There is no difference between BS model and actual values.

Alternative Hypothesis (\( H_a \)): There is significant difference between BS model and actual values.

**DATA ANALYSIS AND INTERPRETATION:**

From the table 1 and 2 it is observed that only in case of Bharti Airtel Call option, there is no significant difference between BSOPM model price and the actual option prices. In other 9 call option and all ten put options BSOPM model values are significantly different from the actual option value.

**SUMMARY, CONCLUSION & RECOMMENDATION:**

It shows that the BSOPM model does not seem fit for the pricing of options in Indian market where the volumes in options are quite uncertain and very less compared to index options. It is suggested that one should also consider the factor like implied volatility and volumes for pricing option contract. Recently, the impact of implied volatility (volatility derived from Black Scholes Formula) is taken into consideration by some of the research. It has been observed that option contract which has a high implied volatility is generally overpriced and the option contract with low implied volatility in underpriced. It is recommended that implied volatility should be taken into consideration for the pricing of option contract.
REFERENCES

**Research Guru: Online Journal of Multidisciplinary Subjects (Peer Reviewed)**

**Table 1**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Call Option</th>
<th>No. Of Observation</th>
<th>Mean Theoretical Price</th>
<th>Mean Market Price</th>
<th>P Value (Paired Comparison)</th>
<th>Hypothesis Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bharti Airtel</td>
<td>682</td>
<td>11.01531883</td>
<td>11.38039648</td>
<td>0.5103**</td>
<td>Null Hypothesis Accepted</td>
</tr>
<tr>
<td>2</td>
<td>HDFC BANK</td>
<td>777</td>
<td>12.35638233</td>
<td>18.1124036</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>3</td>
<td>HDFC</td>
<td>758</td>
<td>13.43209047</td>
<td>22.03720317</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>4</td>
<td>ICICI BANK</td>
<td>1091</td>
<td>28.47331452</td>
<td>39.09813127</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>5</td>
<td>INFOSYS</td>
<td>1118</td>
<td>81.67184563</td>
<td>101.4836453</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>6</td>
<td>ITC</td>
<td>796</td>
<td>9.409983374</td>
<td>10.24792453</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>7</td>
<td>L&amp;T</td>
<td>341</td>
<td>40.25466732</td>
<td>38.13264706</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>8</td>
<td>RELIANCE</td>
<td>670</td>
<td>16.98393992</td>
<td>24.4696562</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>9</td>
<td>TATAMOTORS</td>
<td>707</td>
<td>13.90197374</td>
<td>15.33415424</td>
<td>0.1903**</td>
<td>Null Hypothesis Accepted</td>
</tr>
<tr>
<td>10</td>
<td>TCS</td>
<td>1084</td>
<td>46.24498337</td>
<td>53.79718375</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Put Option</th>
<th>No. Of Observation</th>
<th>Mean Theoretical Price</th>
<th>Mean Market Price</th>
<th>P Value (Paired Comparison)</th>
<th>Hypothesis Testing (difference at 5% level of significance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bharti Airtel</td>
<td>683</td>
<td>13.10894532</td>
<td>14.1558651</td>
<td>0.0016</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>2</td>
<td>HDFC BANK</td>
<td>748</td>
<td>14.10472798</td>
<td>13.04839786</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>3</td>
<td>HDFC</td>
<td>780</td>
<td>29.57846491</td>
<td>22.90333333</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>4</td>
<td>ICICI BANK</td>
<td>924</td>
<td>45.31202314</td>
<td>34.99344529</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>5</td>
<td>INFOSYS</td>
<td>1009</td>
<td>123.7638708</td>
<td>83.20352183</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>6</td>
<td>ITC</td>
<td>738</td>
<td>7.722486276</td>
<td>7.289484396</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>7</td>
<td>L&amp;T</td>
<td>861</td>
<td>431.2553196</td>
<td>32.49181185</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>8</td>
<td>RELIANCE</td>
<td>710</td>
<td>24.51866727</td>
<td>21.19816643</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>9</td>
<td>TATAMOTORS</td>
<td>635</td>
<td>20.86337699</td>
<td>10.56273292</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
<tr>
<td>10</td>
<td>TCS</td>
<td>1084</td>
<td>46.24498337</td>
<td>53.79718375</td>
<td>Less than 0.0001</td>
<td>Null Hypothesis Rejected</td>
</tr>
</tbody>
</table>